[redacted]

Southampton
21st February 1964.

Dear Tom,

At present I don't know whether I am suffering most from a cold which has kept me in bed for the last couple of days, or from the considerable quantities of hot lemon and rum which I have consumed to try to get rid of it. Of course, the cold had to come at a time when I was about to work on a new patch of garden in the Hall grounds, and it prevented me from going to an excellent concert in the university yesterday – but I have had a most welcome rest.

First, a bit about dancing. The dance concocted from Ghivers for the Inter-Varsity folk dance festival was apparently well received. The Southampton people were asked for the instructions – I hope that they did not let anyone have them since the dance is such a fake and did not really hang well together. I started teaching them the fling ten days ago, but to my great surprise I found that I could not reconstruct the set dances from my all too brief notes so I could not start them on any of the MacNab dances. Since there are only six of them the choice of dance is rather restricted

Sometime when I am in Liverpool I'll ~~collate~~ copy the instructions for McLeod of Lewis and McKenzie of Seaforth and check the corrections to the printed instructions for McLaine of Lochbuie if I may. My tape recorder has not yet come back from the factory - I sent it to have the motors adjusted, they were very noisy and the brakes were catching. It should not be long now, so I should be able to return the tapes, and the film strip, before you go away. I have had a letter of introduction from Pat S-S to the people on Foulness Island, and plan to go there sometime at Easter. What a distance it is from here - it certainly can't be a weekend trip.

Talking about your going away, and recalling that I went to see how my Turkish tulip was getting on this morning, I am reminded to enclose a form which sets out the conditions for importing wild plants. I write to them giving ~~places~~ countries, dates, and the specified address to which the plants will be taken (item 2 on form), and they issue a licence. It is certainly worth taking one, even if you don't use it. I shall be writing to them soon for one to take to America with me in summer - I am going on a British Universities North America Club charter flight from July ~~5th~~ 6th to ~~Sept~~ 28th.

And now to mathematics. We seem to be holding our own in Southampton in the struggle for staff. A week ago we appointed two new men, a young Quantum theory man from Sir John Cass, and

Lederer (real variable + analytic topology) from Sheffield. I am going up to look at York on March 11th. My real motive is to see what my architect friends from Liverpool have done to the site, but I shall also enjoy hearing about their academic plans. Everything here is in the melting pot, and we are subject to arguments in favour of all sorts of reforms. Most of the contestants in the fight will no doubt find themselves skirmishing with private opponents while the university as a whole goes on with only minor changes in structure. We have many advocates of 'honours only' and equally many for the Sussex scheme of a general first half year. Since ~~all~~ the various schemes which have been tried here in the last seven years have all foundered on the inability or unwillingness of most lecturers to construct reasonable integrated courses for the non-honours men, I think that any new scheme will follow suite unless one can generate more enthusiasm for pedagogy than we have at present.

Vic Hale wrote to ask me to go to York and talk to their seminar. They have four lecturers, all working in different fields, and there will be some lecturers from the teachers training college and some ~~teach~~ teachers – would I make the first half of my lecture understandable by all of them? Ah, well; I plan to talk about some contributions of Karl Menger to mathematics bringing in his theorem that a convex complete metric space contains with every two points a segment joining them, his concept of statistical metric spaces and his 'Calculus, a modern approach'. I shall be able to look at the audience and adjust the proportions accordingly.

On examining that book again in more detail one becomes aware of strange difference in difficulty in introducing derivatives and integrals. While he gives a reasonable treatment of derivatives with a thorough discussion of domains of functions and extension by continuity, he gives only an outline of an approach to integrating 'simple' functions without defining the word 'simple'. Perhaps that is inevitable at an elementary level.

When we approach Riemann integration at university we have to keep in mind that for elementary functions $\int Df = f$ is a fact which has been used by the students for at least two years. I would not try to discuss the Riemann integral with non-honours students until the problem of investigating $\int Df$ for some non-simple functions arose, probably through an investigation of series of functions. This automatically puts one in a 'uniformity' situation. I think that full explanations of the details of the theory are too difficult for non-honours students, but that at this stage they need something more than the usual half hearted indications, and that ~~your~~ with your approach through step functions it will be easier to give a good account of the steps involved. In particular I feel that one could put across the need to show that $T(f) = \lim_{n\to\infty} S(h_n)$ is independent of the sequence $\{h_n\}$, easier than one can indicate the significance of convergence on the net of subdivisions of $[a,b]$ which occurs in the standard approach. This is probably final year work. One would need to state the properties of the integral earlier (which at any rate they have used) and to declare firmly that the job is too difficult for it to be worth bodging it at that stage.

For honours men one would certainly expect a fairly early treatment of the integral, but perhaps not until 2nd year (your 3rd year). Since they will already have used the properties which are to be proved to be valid, except perhaps the classes of functions involved, one should choose that method of approach which reveals most about mathematical ideas. I would favour a historical approach in the first place, for there is no doubt that the method of exhaustion for simple figures still has great significance, and is a good link with the school approach to area. The problem of extending this technique to more complicated figures and graphs which arose as mathematics develops should then be discussed. It should be made clear that properties like $\int_a^b + \int_b^c = \int_a^c$ and $\int f+g = \int f + \int g$ are to be required of any extended theory – they are not bonuses at the end – if they did not hold we should reject the theory. One could then look at the definition $\inf \sum M_i \delta_i = \sup \sum m_i \delta_i$. It appears to me that in the detailed work which would follow this there arise only two ideas of general application, (i) that a monotone bounded sequence tends to its bound as a limit, with which they should be thoroughly familiar, (ii) that one can define convergence on nets, which is an idea which will not be met again until advanced work on analytic topology. The treatment needs uniform continuity to show that continuous functions are integrable, and uniform convergence to deal with sequences of functions. On the other hand if one left the historical approach at this point and went to the logical approach of extending the definition and properties of integrals from step functions to regulated functions, the details would convey the following ideas, (i) approximations to continuous functions whereas before they only have Taylor series, and so entry into the whole topic of function spaces, (ii) equivalence classes of sequences of step functions, c.f. equivalence classes of sequences of rationals, (iii) extension of definition from simple to more complicated class of functions, independence of representative of equivalence class — relate this to properties of ~~re~~ irrationals and to properties of ~~more~~ general quotient groups, (iv) natural lead in to Lebesgue integral and to one approach to generalized functions. The technique of course requires uniform continuity and uniform convergence, but you get the results on integrals of limits of uniformly convergent sequences immediately.

Following this line of thought, there seems to be no doubt that your approach is superior to the normal one. May I recommend it for use here — with acknowledgements of course? And may I also mention it at York if the need arises? I hope that you will write out the whole thing sometime, even if you decide in the end not to put it into your book.

Now I must try to find an envelope large enough for all this.

Yours

Frank.